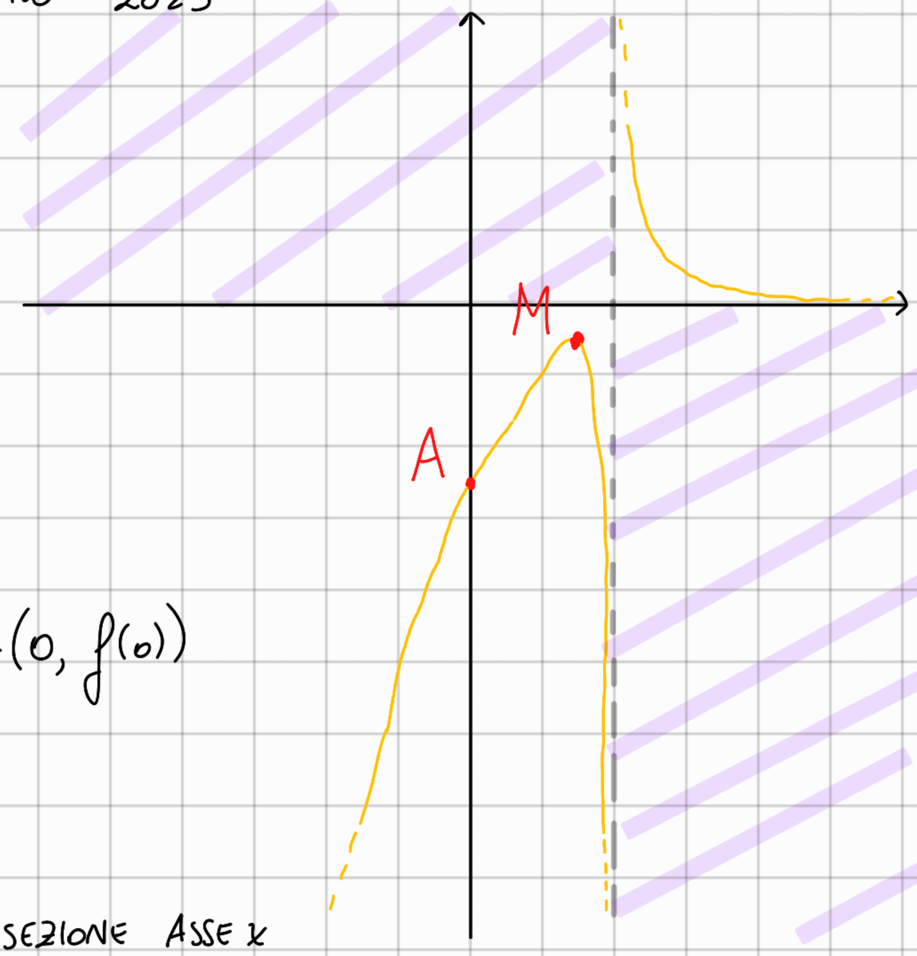
SOLUZIONE ESAME 24 GENNAIO 2025

ESERCIZIO 1

a. $f(x) = \dfrac{e^{3-2x}}{4x-8}$



■ DOMINIO:

C.E.: $4x-8 \neq 0 \to x \neq 2$

$D = (-\infty, 2) \cup (2, +\infty)$

■ INTERSEZIONE ASSE y $\quad A = (0, f(0))$

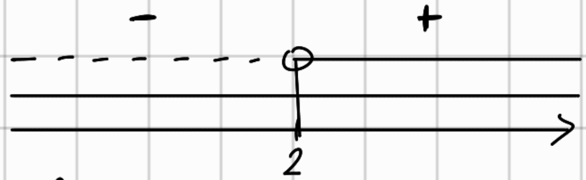
$$f(0) = \frac{e^3}{-8} = -\frac{e^3}{8} \simeq 2.51$$

■ STUDIO DEL SEGNO E INTERSEZIONE ASSE x

$f(x) \geq 0$

$\dfrac{e^{3-2x}}{4x-8} \geq 0$

$e^{3x-2} \geq 0 \quad \forall x \in D \qquad \Big| \qquad 4x-8>0, \quad x>2$



- f è POSITIVA in $(2, +\infty)$
- f è NEGATIVA in $(-\infty, 2)$
- f NON è NULLA MAI $\downarrow$ NO INTERSEZIONI ASSE X

■ LIMITI AGLI ESTREMI DEL DOMINIO e ASINTOTI

$$\lim_{x \to -\infty} \frac{e^{\overbrace{3-2x}^{\to +\infty}}}{4x-8} = \frac{+\infty}{-\infty} \quad F.I.$$

$$\stackrel{D.H}{=} \lim_{x \to -\infty} \frac{-\cancel{2}\, e^{3-2x}}{\cancel{4}\, 2} = -\infty$$

RICERCA AS. OBLIQUO

$$\lim_{x \to -\infty} \frac{e^{3x-2}}{4x^2-8x} = -\infty \to \text{NO AS. OBL.}$$

$$\lim_{x \to +\infty} \frac{e^{3-2x}}{4x-8} = 0 \to \text{ASINTOTO ORIZZONTALE DESTRO} \quad \boxed{y=0}$$

$$\lim_{x \to 2^-} \frac{e^{\overbrace{3-2x}^{\to e^{-1}}}}{4\underbrace{(x-2)}_{\to 0^-}} = \frac{\oplus}{0^-} = -\infty$$

$$\lim_{x \to 2^+} \frac{e^{3-2x}}{4(x-2)} = \frac{\oplus}{0^+} = +\infty$$

$\boxed{x=2}$ ASINTOTO VERTICALE COMPLETO

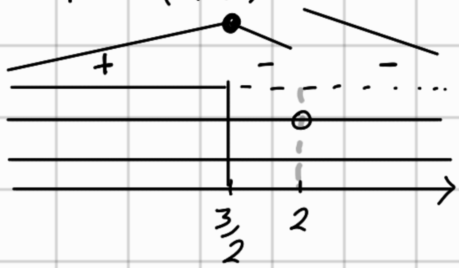
STUDIO DERIVATA PRIMA

$$f(x)=\frac{1}{4}\,\frac{e^{3-2x}}{x-2}$$

$$f'(x)=\frac{1}{4}\,\frac{-2e^{3-2x}(x-2)-e^{3-2x}}{(x-2)^2}=\frac{1}{4}\,\frac{e^{3-2x}(-2x+4-1)}{(x-2)^2}=\frac{1}{4}\,\frac{e^{3-2x}(3-2x)}{(x-2)^2}$$

STUDIO DEL SEGNO $f'(x)\geq 0$

- $\frac{1}{4}e^{3-2x}>0 \rightarrow \forall x\in D$
- $(x-2)^2>0 \rightarrow x\neq 2$
- $3-2x>0 \rightarrow x<\frac{3}{2}$



- f CRESCENTE in $(-\infty,\frac{3}{2})$
- f DECRESCENTE in $(\frac{3}{2},2)\cup(2,+\infty)$
- f STAZIONARIA in $x=\frac{3}{2}$

$$f\left(\frac{3}{2}\right)=\frac{e^{3-\cancel{2}\frac{3}{\cancel{2}}}}{\cancelto{2}{4}\frac{3}{\cancel{2}}-8}=\frac{1}{-2}=-\frac{1}{2}\rightarrow M=\left(\frac{3}{2},-\frac{1}{2}\right)\text{ P.TO DI MASSIMO}$$

STUDIO DERIVATA SECONDA

$$f''(x)=\frac{1}{4}\,\frac{\left[e^{3-2x}(3-2x)\right]'(x-2)^{\cancel{2}}-e^{3-2x}(3-2x)2\cancel{(x-2)}}{(x-2)^{\cancelto{3}{4}}}=$$

$$=\frac{1}{4}\,\frac{\left[-2e^{3-2x}(3-2x)+e^{3-2x}(-2)\right](x-2)-2e^{3-2x}(3-2x)}{(x-2)^3}=$$

$$=\frac{e^{3-2x}\left[(\overbrace{-6+4x-2}^{-8+4x})(x-2)-6+4x\right]}{4(x-2)^3}=\frac{e^{3-2x}\left[-8x+4x^2+16-8x-6+4x\right]}{4(x-2)^3}=$$

$$=\frac{e^{3-2x}(\cancelto{2}{4}x^2-\cancelto{6}{12}x+\cancelto{5}{10})}{\cancel{4}(x-2)}=\frac{e^{3-2x}(2x^2-6x+5)}{2(x-2)^3}$$

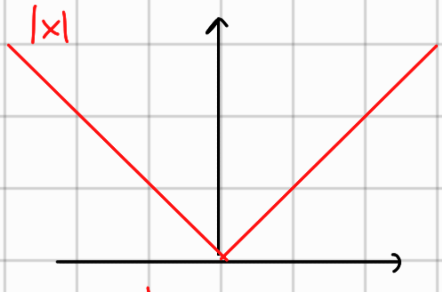
- $\frac{e^{3-2x}}{2}>0 \rightarrow \forall x\in D$
- $2x^2-6x+5\geq 0 \rightarrow \forall x\in D$
  $\Delta=36-40=-4<0$
- $(x-2)^3>0\rightarrow x>2$

- f CONCAVA in $(-\infty,2)$
- f CONVESSA in $(2,+\infty)$
- f NON PRESENTA P.TI DI FLESSO

b. Sia $f: D \to \mathbb{R}$ funzione, $x_0 \in D$ tale che il limite

$$\lim_{h \to 0} \frac{f(x_0+h) - f(x)}{h}$$

esiste finito allora tale limite è detto $f'(x_0)$.



NON E' DERIVABILE IN $x_0 = 0$

C. TEOREMA DI WEIERSTRASS

Sia $f: D \longrightarrow \mathbb{R}$ CONTINUA IN $[a,b] \subseteq D$ allora in tale intervallo essa ammette massimo e minimo assoluti

cioè $\exists\, x_1, x_2$ t.c. $f(x_1) = m$, $f(x_2) = M$ e

$$f(x) \leq M \quad \forall x \in [a,b]$$
$$f(x) \geq m \quad \forall x \in [a,b]$$

$\boxed{\text{ESERCIZIO} \quad 2}$

$$f(x) = \cos(5x) \sin^2(5x)$$

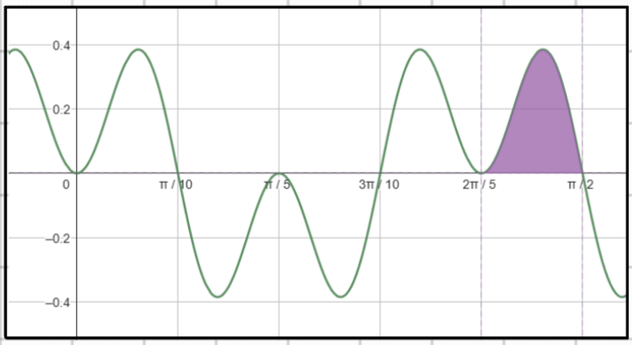
a. $$\int \cos(5x) \sin^2(5x)\, dx = \frac{1}{5} \int \underbrace{5\cos(5x)}_{g'(x)} \underbrace{\sin^2(5x)}_{f'(g(x))} dx = \quad = f(g(x))$$

$$= \frac{1}{5} \frac{\sin^3(5x)}{3} + c = \frac{1}{15} \sin^3(5x) + c \qquad f = \frac{x^3}{3}$$

b. $$\int_{\frac{2}{5}\pi}^{\pi/2} \cos(5x) \sin^2(5x)\, dx = \left[\frac{1}{15} \sin^3(5x)\right]_{\frac{2}{5}\pi}^{\pi/2} = -\frac{1}{15}\left[\sin^3\left(\frac{5}{2}\pi\right) - \sin^3\left(\cancel{5}\frac{2}{\cancel{5}}\pi\right)\right] = \frac{1}{15}[1-0] = \frac{1}{15}$$

$\sin''\left(\frac{\pi}{2}\right) = 1 \qquad \sin(2\pi) = 0$

c.



ESERCIZIO 3

$$v(t) = \frac{t}{2t^2+1} \ m/s$$

Per sapere lo spostamento tra $t=0$ e $t=5$ calcolo

$$\int_0^5 \frac{t}{2t^2+1}\,dt = \frac{1}{4}\int_0^5 \frac{4t}{2t^2+1}\,dt = \frac{1}{4}\ln|2t^2+1|\Big]_0^5 = \frac{1}{4}\ln(2\cdot 25+1) - \frac{1}{4}\ln(2\cdot 0+1)$$

$$= \frac{1}{4}\ln(51) - \cancel{\frac{1}{4}\ln(1)} = \frac{1}{4}\ln(51) \simeq$$

$$\simeq 0.98$$

ESERCIZIO 4

a. Calcolo $\bar{x} = \frac{1}{7}(2.5+6+3+4.5+5.5+7+6.5) = \frac{35}{7} = 5$

$$\bar{y} = \frac{1}{7}(-105-251-119-144-152-224-271) = -\frac{1267}{7} = -181$$

$$\sigma_x^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2 = \frac{1}{7}\left[(2.5-5)^2+(6-5)^2+(3-5)^2+(4.5-5)^2+(5.5-5)^2+(7-5)^2+(6.5-5)^2\right]$$

$$= \frac{1}{7}\left[6.25+1+4+0.25+0.25+4+2.25\right] = \frac{18}{7} \simeq 2.57$$

$$\sigma_{xy} = \frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})(y_i-\bar{y}) = \frac{1}{7}\left[(2.5-5)(-106+181)+(6-5)(-251+181)+\dots+(6.5-5)(-271+181)\right] = \frac{-606.5}{7}$$

OPPURE

$$\sigma_{xy} = \overline{xy} - \bar{x}\bar{y} = \frac{1}{7}\left[2.5(-106)+6(-251)+3(-119)+4.5(-144)+5.5(-152)+7(-224)+6.5(-271)\right] - [5][-181]$$

$$\alpha = \frac{-\frac{606.5}{\cancel{7}}}{\frac{18}{\cancel{7}}} = \frac{\sigma_{xy}}{\sigma_x^2} = -\frac{606.5}{18} \simeq -33.7 \quad ; \quad \beta = \bar{y} - \alpha\bar{x} = -181-(-33.7)(5) = -181+168.4 \simeq -12.5$$

Se $y^* = -180 \Rightarrow y = \alpha x^* + \beta \rightarrow x^* = \frac{y^*-\beta}{\alpha} = \frac{-180+12.5}{-33.7} = \frac{-168.5}{-33.7} \simeq 5$ UI

b. DIFF. GLICEMIA è una variabile numerica continua.

DATI RIORDINATI

-106	-119	-144	$\boxed{-152}$	-224	-251	-271
$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(4)}$	$x^{(5)}$	$x^{(6)}$	$x^{(7)}$

MEDIANA = -152

VARIANZA CAMPIONARIA

$$S_y^2 = \frac{1}{m-1}\sum_{i=1}^{m}(y_i-\bar{y})^2 = \frac{1}{6}\left[(-106+181)^2+(-251+181)^2+(-119+181)^2+(-144+181)^2+(-152+181)^2+(-224+181)^2+(-271+181)^2\right] \simeq 4421$$

Risultati esercizio 4

X	Y	X-X medio	Y-Y medio	(X-X medio)^2	(X-Xmedio)(Y-Ymedio)
2.5	-106	-2.5	75	6.25	-187.5
6	-251	1	-70	1	-70
3	-119	-2	62	4	-124
4.5	-144	-0.5	37	0.25	-18.5
5.5	-152	0.5	29	0.25	14.5
7	-224	2	-43	4	-86
6.5	-271	1.5	-90	2.25	-135

	X	Y				
somma	35	-1267		18		-606.5
media	5	-181	varianza X	2.57	covarianza	-86.64

alpha	beta
-33.694	-12.5278

Valore test	**4.97**	**-180**

mediana Y	-152
varianza campionaria Y	4421.333